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Business process monitoring using a robust Max-Half-Mchart developed with Fast S Estimator

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ABSTRACT

In a business environment, ensuring production processes plays a crucial role in a company's quality and stability. One method that can be utilized to supervise the quality of business processes is the control chart. Control charts are useful tools for quickly monitoring a business process. In practice, multivariate control charts are often preferred because they can evaluate several quality characteristics simultaneously, making them more efficient than monitoring each variable separately. Furthermore, simultaneous multivariate control charts are capable of evaluating process changes in mean and variability. Besides selecting the appropriate method, attention should also be given to the data involved in the business process. Data in business processes can contain outliers that cause classification errors. Therefore, a strong estimator is needed combined with a control chart to be resistant to outliers. The Fast S estimator is recognized as a robust technique that can effectively manage data affected by outliers. The use of the Fast S estimator within the Max-Half-Mchart framework improves the sensitivity of the monitoring scheme toward changes occurring in the production process. Based on the obtained results, the proposed chart generated six signals, whereas the conventional chart produced only two signals. This finding demonstrates a clear difference between robust and non-robust approaches with respect to detection performance. Accordingly, the proposed method shows greater sensitivity than methods that do not incorporate a robust estimator.

Keywords: Fast S; Robust Max-Half-Mchart; business process control; control chart

1. INTRODUCTION

Businesses can remain competitive by continuously innovating and maintaining product quality to meet customer expectations. This requires understanding consumer needs and consistently implementing quality control to ensure the production of high-quality products (Saputra & Renilaili, 2019). In the business world, cement production must also pay attention to the quality of the cement production process, namely whether the production process is still under control. Statistical quality control is an approach to maintaining quality in a production process. In practice, quality data is not limited to a single quality variable, but can be more than one quality variable. Processing data using a single variable separately makes the work ineffective and inefficient. Therefore, multivariate control charts can be utilized as instruments for monitoring multiple quality variables at the same time. The control chart used in previous studies on cement quality is the Shewhart type, which means that the shift in cement quality data is large (Montgomery, 2020). A well-known method in multivariate quality control chart is the Hotelling T^2. This method is designed for situations in which the data satisfy the assumption of multivariate normality and are not affected by outlying observations. However, in real-world situations, quality data can contain outliers, whether from measurement errors, machine failures, or other factors. The presence of these outliers will compromise the results of quality control using control charts.

The presence of this issue has motivated the development of robust estimators determining multivariate statistical parameters of quality data. The outcomes generated by these estimators are commonly incorporated into control chart procedures for monitoring quality characteristics. Because robust estimators are less affected by extreme observations, the resulting control charts are expected to provide more reliable detection performance. One robust estimator frequently applied is Fast S S (Salibian & Yohai, 2006). Fast S represents an extension of the S estimator and is recognized for having a high breakdown point as well as strong resistance to substantial contamination. Consequently, this estimator is suitable for estimating the mean and covariance structure of quality variables.

Quality variables obtained from production activities can be monitored using simultaneous multivariate control charts. These charts are considered effective since they are capable of monitoring mean and variability from several quality variables simultaneously. One method that has gained considerable attention is the Max-Half-Mchart. Other multivariate control chart methods include Max-MCUSUM (Cheng & Thaga, 2005), Max-MEWMA (Xie, 1999), and Max-Mchart (Thaga & Gabaitiri, 2006). In addition, studies related to Max-Half-Mchart have also been conducted by Kruba (Kruba et al., 2021). Related studies on quality monitoring using the Max-Half-Mchart with the Fast MCD and Det MCD were carried out by Sembada et al. (Sembada et al., 2025), while performance evaluation of the control chart was investigated by Ahsan et al. (Ahsan et al., 2026). Previous studies on the Max-Half-Mchart have applied robust estimators such as Fast-MCD and Det-MCD for multivariate process monitoring. The use of these robust estimators has contributed to improving the reliability of simultaneous monitoring of process mean vectors and covariance structures in multivariate quality control. Nevertheless, the application of the Fast S estimator within simultaneous multivariate control chart frameworks has received relatively limited attention in the existing literature.

Fast S is recognized as a robust estimator with several desirable statistical properties. In particular, the estimator possesses a high breakdown point and strong resistance to multivariate outliers, making it less sensitive to contaminated observations. In addition, Fast S provides robust estimation of multivariate location and covariance parameters, which are essential components in the construction of multivariate control charts. These properties indicate that the Fast S estimator may be suitable for multivariate process monitoring, especially in situations where the observed data contain outliers or deviate from standard distributional assumptions. Based on these considerations, this study applies the Fast S estimator within the Max-Half-M chart to monitor cement quality data. The proposed approach is expected to support a more robust monitoring process in detecting shifts in process mean vectors and covariance structures under the presence of outlying observations. Through this application, the study also extends the implementation of the Fast S estimator in simultaneous multivariate control chart for business process monitoring.

2. METHOD

2.1. Max-Half-Mchart

This approach was introduced as an alternative to overcome a number of drawbacks found in the Max-Mchart. A major limitation of the original approach lies in its high dependence on the accuracy of the chi-square cumulative distribution function, especially in situations where the computed probabilities are extremely small or approach zero. Under these conditions, the inverse standard normal distribution may generate very large negative values, causing the Max-Mchart statistic to be strongly influenced by the magnitude of those negative values. As a result, this chart can misclassify processes that are actually still in control and out of control. To address this problem, an alternative approach, the half-normal approach, is used, which can mitigate this effect and provide more stable results. The formula can be expressed as follows (Kruba et al., 2021).

$$M_i^{IH} = \max\{Z^{IH}, V^{IH}\}, i = 2,3, \dots n \tag{1}$$

with

$$Z_i^{IH} = Q^{-1}\left[H_p\{(\boldsymbol{x}_i - \boldsymbol{\mu}_0)'\boldsymbol{\Sigma}_0^{-1}(\boldsymbol{x}_i - \boldsymbol{\mu}_0)\}\right] \tag{2}$$

and

$$V_i^{IH} = Q^{-1}\left[H_p\left\{\frac{1}{2}(\boldsymbol{x}_i - \boldsymbol{x}_{i-1})'\boldsymbol{\Sigma}_0^{-1}(\boldsymbol{x}_i - \boldsymbol{x}_{i-1})\right\}\right] \tag{3}$$

$Q(\cdot)$=the cumulative distribution function for a standard half-normal variable
$H(\cdot)$=cumulative distribution function for a chi-square variable with p degrees of freedom.
The parameters μ_0 and Σ_0 represent the in-control process mean vector and covariance matrix, respectively. The statistic $Z_i^{IH}$is used to monitor shifts in the process mean vector, whereas $V_i^{IH}$is used to detect changes in process variability.

2.2. Fast S

The Fast S-estimator is a robust statistical method used to estimate the multivariate location and covariance matrix in the presence of outliers. The following framework describes the Fast S procedure. At the beginning of the process, The Fast S-estimator formulation described in (Hubert et al., 2015) is presented as follows. Suppose that $\mathbf{x}_1, \dots, \mathbf{x}_n \in R^p$ represent multivariate observations. The multivariate location and scatter estimates are represented by the pair $(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\Sigma}})$, which is determined by minimizing $|\ S\ |$subject to a specified constraint where m denotes the location vector, S is the scatter matrix, $\rho(\cdot)$ is a bounded loss function, b is a predefined constant:

$$\left(\frac{1}{n}\right)\sum_{n=1}^{n}\rho\sqrt{(\boldsymbol{x}_i - \boldsymbol{m})^t\boldsymbol{S}^{-1}(\boldsymbol{x}_i - \boldsymbol{m})} = b \tag{4}$$

To simplify the optimization process, the covariance matrix is decomposed into a scale component and a shape component as follows $\sigma^2\boldsymbol{\Gamma}$, where σ is the scale parameter and $\boldsymbol{\Gamma}$ is the shape matrix satisfying $|\ \boldsymbol{\Gamma}\ | = 1$. The scale parameter is defined as $\sigma =|\ \boldsymbol{\Sigma}\ |^{1/(2p)}$. The aim is to obtain the triple $(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\Gamma}}, \hat{\sigma})$ that yields the smallest value of s while satisfying the following condition:

$$\frac{1}{n}\sum_{i=1}^{n}\rho\left(\frac{\sqrt{(\boldsymbol{x}_i - \boldsymbol{m})^t\boldsymbol{G}^{-1}(\boldsymbol{x}_i - \boldsymbol{m})}}{s}\right) = b \tag{5}$$

for every $(\boldsymbol{m}, \boldsymbol{G}, s)$, where $\in \mathbb{R}^p$, $\boldsymbol{G}$ denotes a symmetric positive definite matrix of order $p \times p$ with $|\boldsymbol{G}| = 1$ and s is a positive scalar.

The resulting multivariate location and dispersion estimates can be expressed as $(\widehat{\boldsymbol{\mu}}, \hat{\sigma}^2\widehat{\boldsymbol{\Gamma}})$. The Fast S begins by constructing several initial estimates, namely $\left(\widehat{\boldsymbol{\mu}}_1^{(0)}, \widehat{\boldsymbol{\Gamma}}_1^{(0)}, \hat{\sigma}_1^{(0)}\right), \dots, \left(\widehat{\boldsymbol{\mu}}_N^{(0)}, \widehat{\boldsymbol{\Gamma}}_N^{(0)}, \hat{\sigma}_N^{(0)}\right)$. These initial values are obtained from N random subsets of size $p+1$ whose covariance matrices have nonzero determinants. Afterward, the mean $\widehat{\boldsymbol{\mu}}_l^{(0)}$ and covariance matrix $\widehat{\boldsymbol{\Sigma}}_l^{(0)}$ for the l-th subset are computed. The next step is to determine for each $l = 1, \dots, N$

$$\widehat{\boldsymbol{\Gamma}}_l^{(0)} = |\widehat{\boldsymbol{\Sigma}}_l^{(0)}|^{-1/p}\,\widehat{\boldsymbol{\Sigma}}_l^{(0)} \tag{6}$$

and

$$\hat{\sigma}_l^{(0)} = \operatorname{med}_{i=1}^{n}\left\{\sqrt{(\boldsymbol{x}_i - \widehat{\boldsymbol{\mu}}_l^{(0)})^t(\widehat{\boldsymbol{\Gamma}}_l^{(0)})^{-1}(\boldsymbol{x}_i - \widehat{\boldsymbol{\mu}}_l^{(0)})}\right\} \tag{7}$$

Next, refinements are made using k-step iterative steps, called I-steps, to obtain:

$$\left(\widehat{\boldsymbol{\mu}}_1^{(k)}, \widehat{\boldsymbol{\Gamma}}_1^{(k)}, \hat{\sigma}_1^{(k)}\right), \dots, \left(\widehat{\boldsymbol{\mu}}_N^{(k)}, \widehat{\boldsymbol{\Gamma}}_N^{(k)}, \hat{\sigma}_N^{(k)}\right)$$

In the j-th I step, refinements are made to the estimates $\left(\widehat{\boldsymbol{\mu}}_l^{(j-1)}, \widehat{\boldsymbol{\Gamma}}_l^{(j-1)}, \hat{\sigma}_l^{(j-1)}\right)$ with the following steps:

1. Updating the scale:

$$\hat{\sigma}_l^{(j)} = \hat{\sigma}_l^{(j-1)}\sqrt{\frac{1}{nb}\sum_{i=1}^{n}\rho\left(\frac{\sqrt{\left(\boldsymbol{x}_i - \widehat{\boldsymbol{\mu}}_l^{(j-1)}\right)^t(\widehat{\boldsymbol{\Gamma}}_l^{(j-1)})^{-1}\left(\boldsymbol{x}_i - \widehat{\boldsymbol{\mu}}_l^{(j-1)}\right)}}{\hat{\sigma}_l^{(j-1)}}\right)} \tag{8}$$

2. Calculating weights:

Use $\hat{\sigma}_l^{(j)}$ to calculate the weights:

$$w_i^{(j)} = \frac{\rho'(u)}{u} \tag{9}$$

$$u = \frac{\sqrt{\left(\boldsymbol{x}_i - \widehat{\boldsymbol{\mu}}_l^{(j-1)}\right)^t(\widehat{\boldsymbol{\Gamma}}_l^{(j-1)})^{-1}\left(\boldsymbol{x}_i - \widehat{\boldsymbol{\mu}}_l^{(j-1)}\right)}}{\hat{\sigma}_l^{(j)}} \tag{10}$$

3. Next, using these weights, the updated location vector $\widehat{\boldsymbol{\mu}}_l^{(j)}$ and $\widehat{\boldsymbol{\Sigma}}_l^{(j)}$ are computed. The corresponding shape matrix is then updated as $\widehat{\boldsymbol{\Gamma}}_l^{(j)} = |\widehat{\boldsymbol{\Sigma}}_l^{(j)}|^{-1/p}\,\widehat{\boldsymbol{\Sigma}}_l^{(j)}$

Following the k-step iteration process (I-step), the scale parameter $\hat{\sigma}_l^{(k)}$ is refined further for each pair $\left(\widehat{\boldsymbol{\mu}}_l^{(k)}, \widehat{\boldsymbol{\Gamma}}_l^{(k)}, \hat{\sigma}_l^{(k)}\right)$ through an iterative solution:

$$\hat{\sigma}_l^{(k+1)} = \hat{\sigma}_l^{(k)}\sqrt{\frac{1}{nb}\sum_{i=1}^{n}\rho\left(\frac{\sqrt{\left(\boldsymbol{x}_i - \widehat{\boldsymbol{\mu}}_l^{(k)}\right)^t(\widehat{\boldsymbol{\Gamma}}_l^{(k)})^{-1}\left(\boldsymbol{x}_i - \widehat{\boldsymbol{\mu}}_l^{(k)}\right)}}{\hat{\sigma}_l^{(k)}}\right)} \tag{11}$$

This procedure continues until convergence is achieved while maintaining $\widehat{\boldsymbol{\mu}}_l^{(k)}$ and $\widehat{\boldsymbol{\Gamma}}_l^{(k)}$ as fixed values. The best estimate is chosen as v=5 (for example): $\left(\widehat{\boldsymbol{\mu}}_1^{(B)}, \widehat{\boldsymbol{\Gamma}}_1^{(B)}, \widehat{\sigma}_1^{(B)}\right), \dots, \left(\widehat{\boldsymbol{\mu}}_v^{(B)}, \widehat{\boldsymbol{\Gamma}}_v^{(B)}, \widehat{\sigma}_v^{(B)}\right)$ which has the smallest scale value after full iteration. Not all $\widehat{\sigma}_l^{(k)}$ for $l = 1, \dots, N$ need to be calculated using the equation above.

For $l > v$, a scale is only calculated if it satisfies:

$$\frac{1}{n}\sum_{i=1}^{n} \rho\left(\frac{\sqrt{\left(\boldsymbol{x}_i - \widehat{\boldsymbol{\mu}}_l^{(k)}\right)^t \left(\widehat{\boldsymbol{\Gamma}}_l^{(k)}\right)^{-1}\left(\boldsymbol{x}_i - \widehat{\boldsymbol{\mu}}_l^{(k)}\right)}}{A}\right) < b \qquad (12)$$

where A denotes the maximum value v among the best scale values obtained so far. The candidate estimates $\left(\widehat{\boldsymbol{\mu}}_1^{(B)}, \widehat{\boldsymbol{\Gamma}}_1^{(B)}, \widehat{\sigma}_1^{(B)}\right), \dots, \left(\widehat{\boldsymbol{\mu}}_v^{(B)}, \widehat{\boldsymbol{\Gamma}}_v^{(B)}, \widehat{\sigma}_v^{(B)}\right)$ are chosen based on the minimum scale values and iteratively refined using I-steps until convergence is reached. At the end of the procedure, the estimator $\left(\widehat{\boldsymbol{\mu}}^{(F)}, \widehat{\boldsymbol{\Gamma}}^{(F)}, \widehat{\sigma}^{(F)}\right)$, is chosen as the final result because it provides the lowest scale value among all candidate estimates. The final Fast S location estimator is denoted by $\widehat{\boldsymbol{\mu}}_{Fast\ S}$, and the corresponding Fast S covariance matrix is defined as $\widehat{\boldsymbol{\Sigma}}_{Fast\ S} = \left(\widehat{\sigma}^{(F)}\right)^2 \widehat{\boldsymbol{\Gamma}}^{(F)}$.

2.3. Proposed Chart

To improve robustness against the presence of outliers, the proposed control chart employs the Fast S approach to obtain location and dispersion estimates within the observed data. In multivariate process monitoring, the presence of outliers can substantially affect parameter estimation and may lead to distorted monitoring results. By reducing the sensitivity of the estimation process to contaminated observations, the Fast S estimator is able to produce more stable parameter estimates. As a result, the estimated covariance structure is expected to better reflect the actual condition of the monitored process.

The integration of the Fast S estimator into the Max-Half-Mchart is intended to support more reliable simultaneous monitoring of process mean vectors and covariance structures. Since the construction of multivariate control charts depends heavily on the estimation of location and covariance parameters, the use of robust estimates can reduce the effect of outlying observations on the resulting monitoring statistics.

By reducing the sensitivity of the estimation process to contaminated observations, the Fast S estimator is able to produce more stable parameter estimates. The main advantage of this estimator lies in its ability to maintain reliable performance even when the dataset contains abnormal observations. Unlike classical estimators, which are highly influenced by extreme values, the Fast S approach focuses on obtaining estimates that are less affected by deviations from the general data pattern. As a result, the estimated covariance structure is expected to better reflect the actual condition of the monitored process. This is particularly important in multivariate monitoring because the relationship among quality characteristics is as meaningful as the individual behavior of each variable.

The integration of the Fast S estimator into the Max-Half-Mchart is intended to support more reliable simultaneous monitoring of process mean vectors and covariance structures. Since the construction of multivariate control charts depends heavily on the estimation of location and covariance parameters, the use of robust estimates can reduce the effect of outlying observations on the resulting monitoring statistics. In this context, the proposed chart is expected to provide a more accurate representation of process stability, especially when the historical data used to estimate parameters are not completely free from contamination.

The use of robust estimation can improve the practical applicability of the Max-Half-Mchart in real industrial settings, where data are rarely ideal. Production processes, service operations, and laboratory measurements often contain irregular observations that cannot always be removed easily. Therefore, a monitoring method that can tolerate such imperfections is highly valuable. By incorporating the Fast S estimator, the proposed control chart becomes less dependent on strict normality and clean-data assumptions. This allows practitioners to monitor complex processes with greater confidence.

The proposed robust Max-Half-Mchart offers a methodological improvement by combining simultaneous monitoring capability with resistance to outliers. This approach is expected to enhance detection accuracy, reduce false alarms, and provide more dependable decision support for quality control applications.

Suppose that $\mathbf{x}_n = \{\mathbf{x}_1, \dots, \mathbf{x}_n\}$ denotes of n-dimensional observations. The corresponding monitoring statistic is then defined as follows:

$$M_{Fast\ S}^{IH} = \max\{Z_{Fast\ S}^{IH}, V_{Fast\ S}^{IH}\}, i = 2,3, \dots n \tag{13}$$

with

$$Z_{Fast\ S}^{IH} = Q^{-1}\left[H_p\{(\boldsymbol{x}_i - \widehat{\boldsymbol{\mu}}_{Fast\ S})' \widehat{\boldsymbol{\Sigma}}_{Fast\ S}^{-1} (x_i - \widehat{\boldsymbol{\mu}}_{Fast\ S})\}\right] \tag{14}$$

and

$$V_{Fast\ S}^{IH} = Q^{-1}\left[H_p\left\{\frac{1}{2}(\boldsymbol{x}_i - \boldsymbol{x}_{i-1})' \widehat{\boldsymbol{\Sigma}}_{Fast\ S}^{-1} (\boldsymbol{x}_i - \boldsymbol{x}_{i-1})\right\}\right] \tag{15}$$

Here, $\widehat{\boldsymbol{\mu}}_{Fast\ S}$ and $\widehat{\boldsymbol{\Sigma}}_{Fast\ S}$ denote the robust location and covariance estimates obtained from the Fast S-estimator. In designing statistical control charts, the commonly used control limits are ±3σ. Based on this value, an Average Run Length (ARL_0) of approximately 370 is obtained, which indicates that on average a false signal will appear once in every 370 observations. Thus, the use of 3σ control limits indirectly produces an ARL_0 of approximately 370, so the limits (UCL) used in this study are the limits that produce an ARL_0 of 370 (Montgomery, 2020).

2.4. Algorithm Of The Proposed Control Chart

The following steps describe the process of computing the monitoring statistics and developing the Fast S-based proposed chart.

Algorithm: Procedure for Estimation and Construction:

Step 1. Prepare the dataset to be analyzed, either simulated multivariate data or real company data.

Step 2. Compute the Fast S estimator statistic.

Step 3. Determine the $\text{M}_{\text{Fast S}}^{\text{IH}}$ statistic.

Step 4. Establish the UCL for the control chart.

Step 5. Construct the control chart by plotting the $\text{M}_{\text{Fast S}}^{\text{IH}}$ statistic values.

Step 6. Compare the calculated statistic with the upper control limit:

1. If $\text{M}_{\text{Fast S}}^{\text{IH}} > \text{UCL}_{\text{Fast S}}$, the process is regarded as having a shift in the mean.
2. If $\text{M}_{\text{Fast S}}^{\text{IH}} > \text{UCL}_{\text{Fast S}}$, the process is regarded as having a change in variability.
3. When both conditions occur simultaneously, it indicates concurrent changes in both the process mean and variability.

3. RESULTS

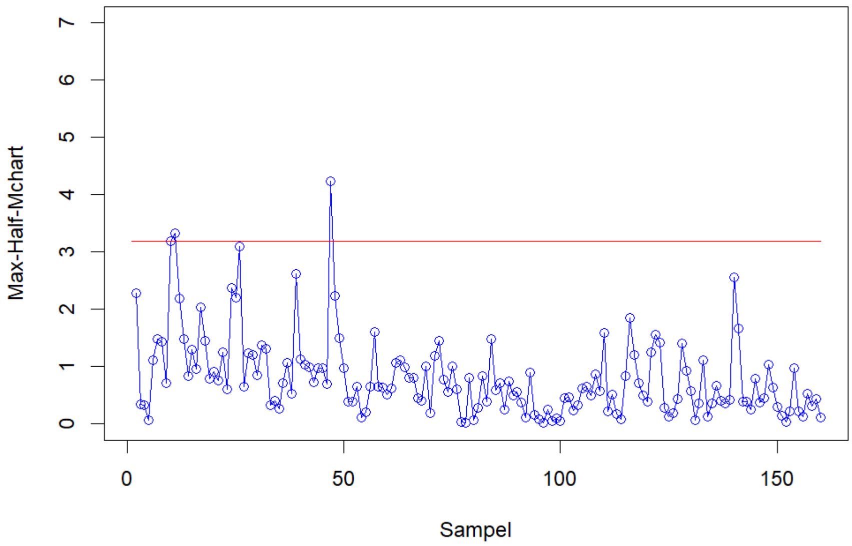

Figure 1. Max-Half-Mchart

Figure 1 presents the monitoring performance generated by the Max-Half-Mchart when applied to production observation data. The chart was employed to assess five cement quality characteristics simultaneously by evaluating changes in the process. The UCL was established at 3.185 because this value produced an ARL_0 close to 370, which corresponds to the 3sigma. Based on the evaluation of 160 observations, two points, specifically the 11th and 47th observations, were found to exceed the control limits. Observations located beyond the control limits indicate that the production process is not operating under statistical control.

Further investigation showed that the 11th observation exceeded the control limit due to a change in the process mean, indicating that the observation deviated from the expected central pattern. Such a condition reflects the possibility of process disturbances that could affect product quality. Likewise, the 47th observation was also identified as out of control because of a change in the process mean.

Based on the evaluation of 160 observations, two points, specifically the 11th and 47th observations, were found to exceed the control limits. Observations located beyond the control limits indicate that the production process is not operating under statistical control. In the context of cement production, this condition is important because cement quality is determined by several interrelated characteristics. A disturbance in one quality characteristic may affect the overall consistency and performance of the final product. Therefore, the detection of out-of-control points should not be ignored, even when only a small number of observations exceed the control limit.

Further investigation showed that the 11th observation exceeded the control limit due to a change in the process mean, indicating that the observation deviated from the expected central pattern. Such a condition reflects the possibility of process disturbances that could affect product quality. The shift in the process mean may be associated with changes in raw material composition, variation in the burning process, differences in grinding conditions, equipment instability, or measurement-related errors. Since cement production involves continuous and highly controlled operations, even small deviations in the process mean can indicate that the process has moved away from its desired operating target.

Likewise, the 47th observation was also identified as out of control because of a change in the process mean. This finding suggests that the abnormality was not caused by random variation alone, but by a special cause that temporarily affected the production process. The occurrence of two out-of-control observations within the monitoring period shows that the process generally remained stable, but certain production stages still require further attention. Identifying the source of these deviations is necessary to prevent similar problems from occurring in future production cycles.

Overall, the results indicate that the proposed chart is capable of detecting abnormal changes in multivariate cement quality characteristics. The ability to identify mean shifts in the 11th and 47th observations demonstrates that the chart can provide useful early warnings for process evaluation. Therefore, the monitoring results can support quality control personnel in taking corrective actions, such as checking raw material inputs, recalibrating equipment, reviewing operational settings, and strengthening process supervision. This makes the chart useful not only as a statistical monitoring tool but also as a practical decision-support instrument for maintaining cement quality consistency.

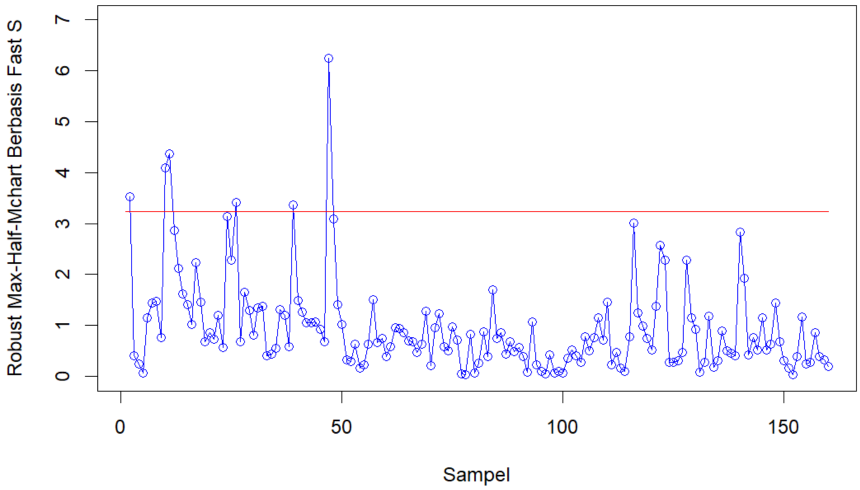

Figure 2. Robust Max-Half-Mchart Developed with Fast S

Figure 2 shows the monitoring results produced by the proposed control chart. The method was used to assess cement production quality through simultaneous observation of five quality characteristics by evaluating the process of the multivariate data. The upper control limit was determined to be 3.23 because this threshold yielded an ARL_0value approximately equal to 370, which follows the 3sigma. Among the 160 observations evaluated, several data points, namely observations 2, 10, 11, 26, 39, and 47, were located above the control limit. These signals indicate that the production process was not operating under stable statistical conditions

A more detailed examination revealed that observation 2 exceeded the control limits due to an increase in process variability, indicating a possible alteration in the data distribution pattern. Meanwhile, observations 10 and 11 were identified as out-of-control points because of shifts in the process mean, which could indicate an issue in the production process. Observation 26 also indicated variability changes, whereas observations 39 and 47 were associated with deviations in the process mean. The findings indicate that the proposed monitoring scheme can recognize various types of process variation. In addition, the chart developed in this study provides more effective detection capability than the conventional chart, especially in situations involving the presence of outlying observations in the dataset.

However, the additional detections obtained using the Fast S estimator should be interpreted with caution. A larger number of detected signals may indicate higher sensitivity in identifying potential process deviations. Therefore, the results of this study are limited to the identification of out-of-control observations as an indication that the business process may not be operating properly, thereby allowing further evaluation of the production process. In addition, the conclusions of this study are restricted to the observed multivariate cement quality data and the process conditions considered in this application.

These findings are generally consistent with previous studies on robust multivariate control charts, which reported that robust estimators tend to reduce the masking effect caused by outliers and produce monitoring statistics that are less influenced by extreme observations. In the present study, similar behaviour was observed in the proposed method, where more process deviations were identified in multivariate cement quality data compared with the classical approach. The proposed method detected six out-of-control observations, which was the same number identified by the Det-MCD estimator, whereas the Fast-MCD approach detected only two out-of-control observations in the previous study. These results suggest that the Fast S estimator is more responsive to deviations in multivariate process data. From a robust estimation perspective, this behaviour may occur because robust estimators reduce the influence of outlying observations during the estimation of multivariate location and covariance parameters, allowing process deviations to appear more clearly in the monitoring statistics.

Table 1. Out-of-Control Signal Detection Obtained from Each Monitoring Method

Method	Signals	Mean Shift	Variability Shift
Max-Half-Mchart	2	2	-
Robust Max-Half-Mchart Developed with Fast S	6	4	2

Table 1 provides a comparative overview of the conventional and robust monitoring methods applied to cement quality data. The comparison focuses on the number of points exceeding the control limits, shifts in the central tendency, and variations in process dispersion. Based on the results, the Max-Half-Mchart detected two observations beyond the control limits.

In contrast, the proposed method with robust estimator provided more comprehensive detection results by identifying six observations beyond the control limits. Of these observations, four were associated with mean shifts and the remaining two reflected variability changes. These findings show that the Fast S incorporated method is capable of detecting a wider range of process changes associated with both process mean and variability. Better monitoring performance is achieved by proposed chart in comparison with the conventional chart.

This finding suggests that the robust estimation mechanism of Fast S is more sensitive to deviations in multivariate process data. In conventional estimation methods, extreme observations may affect the estimation of location and covariance parameters, causing certain process shifts to be less apparent in the monitoring statistics. In contrast, the Fast S estimator reduces the influence of outlying observations during parameter estimation, allowing the monitoring statistics to better reflect the overall process behaviour. Consequently, observations that differ substantially from the dominant process pattern can be identified more clearly, resulting in a larger number of detected out-of-control signals in the monitoring of multivariate cement quality data.

The ability of the proposed method to detect both mean shifts and variability changes is particularly important in multivariate quality control. In cement production, quality characteristics are not independent from one another, because changes in one variable may influence or be associated with changes in other variables. For example, variation in chemical composition, fineness, setting time, or strength-related characteristics may occur simultaneously due to changes in raw materials, processing temperature, grinding conditions, or operational settings. Therefore, a control chart that only focuses on mean shifts may fail to capture important changes in process dispersion. By identifying variability changes, the proposed chart provides additional information about the stability and consistency of the production process.

The detection of four observations related to mean shifts indicates that certain observations deviated from the expected central tendency of the monitored quality characteristics. Such deviations may reflect temporary process disturbances, changes in material composition, or operational inconsistencies. Meanwhile, the two observations associated with variability changes indicate that the spread or correlation structure among the quality variables also experienced abnormal behaviour. This is important because increased variability may signal that the process is becoming less predictable, even if the average value remains close to the target. In practical quality control, variability changes should be addressed carefully because they can reduce product uniformity and increase the probability of producing items that do not meet specifications.

Compared with the conventional chart, the proposed robust chart provides a more informative diagnostic result. The conventional chart detected fewer out-of-control observations, which may suggest lower sensitivity when the dataset contains outliers or contaminated observations. This limitation can occur because classical estimators are strongly affected by extreme values. When outliers distort the estimated covariance matrix, the control region may become wider or less representative of the actual process condition. As a result, some abnormal observations may still fall within the control limits and remain undetected. The Fast S estimator helps overcome this problem by producing more reliable estimates of location and dispersion, thereby improving the accuracy of the monitoring statistics.

From an operational perspective, the proposed method offers practical benefits for quality control decision-making. By detecting more out-of-control signals, the chart can help quality engineers identify potential problems earlier and investigate their possible causes. Corrective actions may include reviewing raw material inputs, checking equipment calibration, inspecting production parameters, and evaluating laboratory measurement procedures. Early detection is essential because small process deviations can develop into larger quality problems if they are not addressed promptly.

Therefore, robust methods are regarded as more appropriate for statistical quality monitoring when the observed data contain outliers. The integration of the Fast S estimator into the proposed chart strengthens the reliability of multivariate process monitoring by reducing the impact of abnormal observations on parameter estimation. Overall, the results confirm that the proposed robust chart provides better detection capability, broader diagnostic information, and stronger practical relevance for monitoring cement quality characteristics.

This finding suggests that the robust estimation mechanism of Fast S is more sensitive to deviations in multivariate process data. In conventional estimation methods, extreme observations may affect the estimation of location and covariance parameters, causing certain process shifts to be less apparent in the monitoring statistics. In contrast, the Fast S estimator reduces the influence of outlying observations during parameter estimation, allowing the monitoring statistics to better reflect the overall process behaviour. Consequently, observations that differ substantially from the dominant process pattern can be identified more clearly, resulting in a larger number of detected out-of-control signals in the monitoring of multivariate cement quality data. Therefore, robust methods are regarded as more appropriate for statistical quality monitoring when the observed data contain outliers.

4. CONCLUSION

Based on the analysis and comparative evaluation, the proposed method demonstrated superior performance compared with the conventional chart. This was evidenced by its ability to identify a greater number of out-of-control signals, with the proposed approach detecting six cases, whereas the conventional chart identified only two cases. This difference indicates that the robust approach has greater sensitivity in detecting process deviations. The detection of out-of-control signals may indicate the presence of assignable causes and therefore warrants further investigation.

Furthermore, the advantage of the proposed method lies not only in its ability to detect shifts in the process mean but also in its capability to identify changes in process variability. The results showed that the proposed method detected four cases of mean shifts and two cases of variability shifts, whereas the conventional Max-Half-Mchart detected only two mean shifts and failed to identify any variability shifts. These findings demonstrate that the robust approach is capable of simultaneously monitoring changes in both the process mean and variability. This advantage is closely associated with the use of the Fast S estimator, which provides robust parameter estimates in the presence of outlying observations.

Beyond the comparative results, the findings highlight the potential value of incorporating robust covariance estimation into simultaneous multivariate control charts for process monitoring under non-ideal data conditions. From a practical perspective, the proposed method offers an alternative for monitoring industrial processes involving multiple correlated quality characteristics, particularly when outlying observations may affect conventional parameter estimates. The presence of out-of-control observations indicates departures from normal process behavior and may signal the existence of assignable causes related to equipment performance, operator actions, environmental conditions, or other process-related factors. By improving the robustness of parameter estimation, the proposed approach may provide more reliable monitoring results in such situations. Despite these advantages, the method has several limitations. The Fast S estimator is obtained through an iterative optimization procedure, which may require greater computational effort than conventional estimation techniques. Nevertheless, the proposed approach has the potential to be applied to a wide range of industrial processes involving multiple correlated quality characteristics.

Ethical Approval

Not applicable.

Informed Consent Statement

Not applicable

Authors' Contributions

APSR – Conceptualization, literature review, data analysis, writing, corresponding author. MA – Conceptualization, formal analysis, resources, supervision, writing. SWT – Data collection, validation, visualization, writing – review and editing. MGSW– Data collection, data analysis, validation. NA – Methodology, formal analysis, writing – review and editing

Disclosure Statement

The authors declare no conflict of interest.

Data Availability Statement

Due to privacy and ethical considerations, the data are not publicly available.

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